Review article

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MATHEMATICAL DETERMINATION IN NATURE-THE GOLDEN RATIO

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Although the deterministic equation of life predicted and recognized by Albert Einstein is not yet defined, there are clear marks and signs of mathematical regularity which appear in nature showing fascinating accuracy. One of those "God's imprints" in the world that surrounds us is the Fibonacci Sequence, which represents more likely a starting point in revealing certain universal formula of life. The main idea of this paper is to enlighten two points: the Fibonacci sequence and the Golden Ratio, as well as to stress out the importance of reveling the perfect mathematical patterns that exist both in the organic as well as in the inorganic world. Thus, we might be able to understand the physical, spiritual and mental aspect of a human nature more clearly and completely.

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Introduction

"Mathematics is the language in which God has written the Universe."
(Galileo Galilei)

Many early civilizations, such as Chinese, Islamic and Indian gave certain contribution to the development of mathematics. However, it was the ancient Greeks who established the basic principles of mathematical science as it is today. Among the greatest are Euclid with his field of geometry, Archiedes with the approximation of number and Ptolemy who constructed the exact mathematical model which contained all of the celestial bodies gravitating around the Earth. Ptolemy believed that the movements of celestial objects could be explained by certain mathematical terms, which he explained in his scientific piece of work entitled as "The Mathematical Collection." The well-known Euclid's axioms are as important today as they were when he recognized them over 2,000 years ago. Going as far as 228 B.C, and with only basic postulates of trigonometry, the astronomer Eratosthenes estimated the Earth's diameter with over 99 percent accuracy. The Greek discoveries are timeless (1).

Ancient Greeks were strongly convinced that the Universe was created with mathematical accuracy and they mostly applied mathematical principles to static objects. They measured angles of solid objects and calculated their volumes. Also, they widely used mathematics in philosophical purposes. For anyone who wanted to enter his famous Academy, yet was not interested in mathematics, Plato said: "He is unworthy of the name of man who is ignorant of the fact that the diagonal of a square is incommensurable with its side". It has remained as such ever since.

Most of the 17th century, Galileo Galilei put in the center of scientific attention some characteristics of the Nature that could possibly be measured, such as certain variables: weight and force, space and time, acceleration and velocity. Using these results, Galileo was able to establish mathematical equations which described phenomena more precisely and more clearly that had ever been possible before. He said:

"*Nature is written in that great book … I mean the universe, but we cannot understand it if we do not first learn the language and grasp the symbols in which it is written. the book is written in mathematical language, and the symbols are triangles, circles and other geometrical figures, without whose help it is impossible to comprehend a single word of it; without which one wanders in vain through a dark labyrinth.*"

Having in mind that the same mathematical law can explain multiple phenomena, scientists are able to discover relations among those phenomena

that might otherwise go undetected. For example, trigonometric functions could be applied to all wave motions: sound, light and radio waves, as well as waves in gas, in water and many other types of different wave motions. Mathematics can detect various patterns, designs and shapes in Nature left to us to be discovered (2).

Since the mathematical structures of each aspect of Nature, from celestial objects to plants are evident, we need to ask ourselves: "Does the Universe have a specific mathematical construction?" In addition to this idea, the mathematician Mario Livio studied the Nature and its shapes in his book "Is God a mathematician?" Many questions naturally were asked, such as: "Have we humans invented all of mathematics or it had already been out there, waiting to be revealed?" At this point, it is obvious that mathematically defined forms and shapes exist in the smallest as well as in the largest parts of Nature, in living and also in non-living objects. One of those revealing, hidden, perfect patterns of beauty, function and order is the Golden Ratio (3, 4).

Fibonacci Sequence and the Golden Ratio

Golden ratio (Divine proportion, Golden intersection) **ϕ** is the irrational number /2. It means that it cannot be expressed as a fraction and its approximate value is estimated to 1, 6180339887… The intellectual elite of wide domains of scientific interests has been deeply fascinated by the Golden ratio for more than 24 centuries. In fact, it is practically impossible to specify all of the organic and inorganic living structures where this extraordinary number is hidden. Furthermore, according to the scientific point of view, new aspects of the living nature in which the pattern of the gorgeous mathematics have appeared and are still being revealed. There are specific, precise and accurate relations among the existing forms in organic and inorganic aspects of nature which seem to be in perfect proportion and therefore they are called "Divine". In order to be attributed as "Divine", the proportion needs to have specific symmetry, beauty and harmony. It has to show unique specific proportion between the parts and the whole. In order to clarify this, let us assume that we have a line segment divided into two parts **a** and **b**. Divine (golden) ratio is such relation satisfying: the longer part divided by the smaller part equals the whole length of the given line segment divided by the longer part. Expressed mathematically the Golden proportion is determined as:

$$(\mathbf{a} + \mathbf{b})/\mathbf{a} = \mathbf{a}/\mathbf{b} = \boldsymbol{\phi},$$

where a denotes the longer part, and **ϕ** denotes the Golden number (Figure 1).

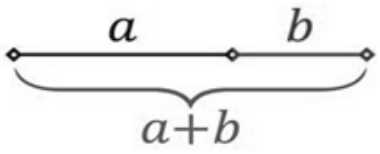

Figure 1. Golden ratio on the line

Pythagoras together with his followers constructed a regular pentagon based on the knowledge of the Golden intersection. They called it Health (Higias) and strongly believed that it represented a pure mathematical perfection. They connected health of the human body directly with the mathematical harmony of the Golden intersection. The Roman statesman, philosopher and mathematician Boethius (480-524 A.D.) showed that a soul and a body follow the same set of mathematical laws of proportion that governs the Cosmos itself.

The sequence of numbers that hides the Golden proportion given as: 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, has been described in the 12th century by the Italian mathematician from Pizza, Leonardo Fibonacci. Each member of this sequence may be calculated as the sum of the two previous members. A very interesting fact of the Fibonacci sequence is that (except for the first few numbers) the quotient of any member of the sequence with its previous one is a constant which tends to 1,6. Thus, we have next equalities: 5/3 = 1,66, 8/5 = 1,6, 13/8 = 1,6, 21/13 = 1,6… Further on, we have that 89:55 = 1,618, rounded to 3 decimal places. Investigating these interesting relations, Fibonacci revealed the sequence in various physical, chemical and biological phenomena. The fascinating moment is that we obtain a sequence of quotients which converges exactly to the Divine number **ϕ**. As the legend says Fibonacci came to this discovery by observing the behavior of rabbits, whose reproduction followed the dynamics of this particular sequence. Fibonacci numbers and the Golden ratio may be found as a pattern in almost every aspect of Nature (4).

Golden shapes

First let us analyze a shape which we are all familiar with. It is a spiral commonly seen in shells. For instance, we can focus on the Chambered Nautilus (Nautilus pompilius Linnaeus, 1758). It is probably the clearest example we could observe. As it grows, the spiral gets longer but it preserves its identical form. Its shape never changes because of the fact that it grows in the path of a spiral that is equiangular and logarithmic.

An amazing fact is that we may present Fibonacci sequence geometrically by drawing specific squares. We begin with 2 squares, with the side

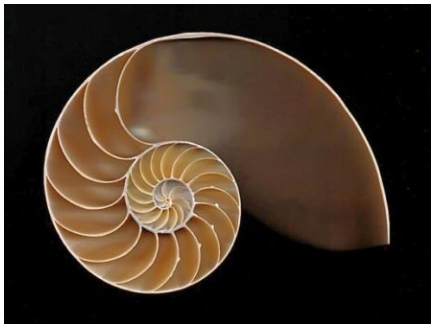

Figure 2. A cross-section displaying the rationed compartmentalization of the *Nautilus pompilius.* Source: tumblr

lengths 1 and draw them side by side. Then, above those two squares we draw a square with the side length 2 (1 + 1 = 2). We continue the procedure by drawing a square of the side length 3 (1 + 2 = 3), right to the obtained rectangle. By adding new squares on the picture, such that the side length of a new square equals the sum of the side lengths of two previous squares, we obtain squares with the side lengths equal to the Fibonacci numbers. Now, if we draw in the obtained squares specific arcs of a circle, we obtain the Golden spiral (Figure 3).

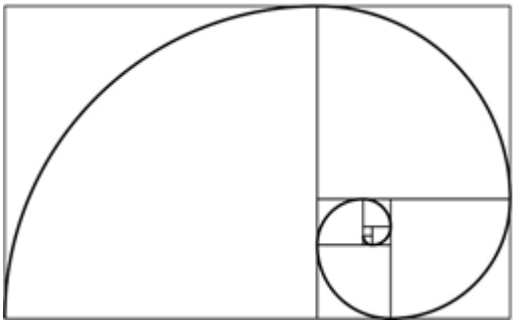

Figure 3. Golden spiral

This spiral follows a precise mathematical pattern. The perfection and beauty of this form we can find in nature approximating the shapes of various natural phenomena extremely well. Apart from shells, we find the Golden spiral in most diverse things, such as: the cochlea of the human ear, spiral seeds, growing fern leaves, ram's horn, sea-horse tail, tornados, the structure of DNA molecule, galaxies. Also, the Milky Way has a number of spiral arms, each of them having a 12 degrees of a logarithmic spiral roughly. The shape of the spiral is identical to the Golden one. Furthermore, the Golden rectangle can be drawn over any spiral galaxy (5, 6). There are numerous examples where this shape may be recognized: in whirlpools, in the tail of a comet as it winds around the sun, in the proportions of the face, in sunflower seed patterns, in the rhythm of heart beats. Really, by looking carefully at a sunflower you will detect two sets of spirals spiraling in opposite directions (rows of seeds or florets). Further on, daisies, dandelions and the ears of most mammals show the same pattern. The fascinating fact is that in the population of bees the ratio of the number of females and the number of males is always ϕ (6).

The length of human fingers, each part from the tip of the base to the wrist is larger than the previous one roughly by the ratio of ϕ. The measurement of the human navel to the floor and the top of the head to the navel presents the Golden ratio. Nevertheless, we are not the only examples of the Golden proportion in the animal world; starfish, dolphins, sea urchins, ants and honeybees also exhibit this proportion. The number of petals on some flowers follows the Fibonacci sequence. In the Darwinian processes it is believed that each petal is placed to allow for the best possible exposure to sunlight and other factors that are necessary for its existence (7).

Golden Ratio in Art, Beauty and Human body

Historically speaking, this specific number can be seen in many ancient architectural creations, such as the Parthenon and the Great Pyramid (8, 9). One has to wonder why Phidias as well as many of Greek sculptors and others in ancient Greece and Egypt use to apply this Golden ratio in designing many of their works of art. The answer may be found in the fact that this ratio seems to be remarkably attractive to the human eye. It also produces what is called a Golden rectangle. Take that the short side of the rectangle is 1, then the long side is 1.618. This rectangular shape is similar to the pattern that has been used in designing the famous Parthenon of Greece and in many other purposes, such as: vases, artistic pictures, windows, doorways and statues. The Golden ratio was used to reach balance and beauty in numerous Renaissance paintings and sculptures. Da Vinci himself applied the Golden ratio in order to define the proportions in his Last Supper, including the dimensions of the walls, backgrounds and the table. This ratio can also be seen in da Vinci's Vitruvian Man and the Mona Lisa. Other artists who also implemented the Golden ratio into their work are: Michelangelo, Rembrandt, Raphael, and Salvador Dali. The United Nations building is an example of Golden rectangle (9). When we talk of the Modern era we could recognize the approximate principle of the same Golden rectangle idea such as: postcards, credit cards, light switch plates, playing cards and writing pads. What makes the Golden proportion so unique? The fact is that art forms can be either of dynamic or static symmetry. In dynamic symmetry it is the proportioning of the areas that is being emphasized which implies changes and power movement, whereas in static symmetry the lines have definite measurements. Dynamic gives life and animation to a work of art, while the effect of stillness and quietness is the expression of static symmetry. This is the meaning of the Golden proportion.

Golden ratio, also called the "Divine proportion" establishes the standard of beauty. Physical attraction among humans depends on ratio (10, 11).

We are attracted to another person's body if that body is in certain proportion. If a face is in the proportion, it is more likely that we will notice it and find it beautiful. The results of recent studies have shown that when tested subject is being exposed to the photos of various random faces, the ones they seem most attractive are those with solid similarities to the Golden ratio. Faces qualified as the most attractive show proportions the width of the face compared to the width of the eyes, nose and eyebrows. All these make the Golden ratio. Even when being seen from the side, the human head represents the Divine proportion (11).

Scientists claim that proportional bodies seem to be healthy. This idea can be seen in the famous image showing a perfectly proportioned human body within a square and a circle, Leonardo da Vinci's Vitruvian Man presenting the Golden ratio in body dimensions: foot to navel: navel to head shows the Golden ratio. Other best known examples of human structure and physiology are: fingers, DNA molecules (a DNA molecule measures 21 angstroms by 34 angstroms at each full cycle of the double helix spiral; in Fibonacci series 21 and 34 are successive numbers). Obviously, human genomic structure is strongly associated with Fibonacci Series and Golden proportions (12, 13). Furthermore, Yamagishi et al. supported the evidence by revealing the existence of Fibonacci Series over whole genomic structure (12, 13, 14): fingerprint, human face and bronchial structure in the lungs. Even more subtle, divine aesthetics might also manifest itself in the essence of a wide variety of inner organ systems. Based on the central visual dynamics, Elliot and his team demonstrated the potential connection between aesthetic preference and Golden ratio. Really, they showed that Golden sectioning in brain activities probably have an impact on the efficiency of visual processing. Golden ratio has also been an object of interest in gynecology: mean values of length/width ratio were reported to have an inverse correlation with age and number of gravidity in a retrospective study investigating ultrasonographic (USG) measurements of non-pregnant uteri. Even more surprisingly, this ratio happened to be 1.618 at peak of fertility (for the age of 21), exactly in concordance with the Golden ratio. These results may suggest that divine aesthetics is more likely to show itself in organ systems at their full capacity and the peak of functioning confirming the measures of divine reflection.

Cardiovascular system might also be a major predilection site of divine aesthetics as measured with Golden ratio and its derivated forms (15). The ancient knowledge related the human heart with the spiritual and esoteric components of human nature including human soul. The results are published in large number of studies associating these concepts with anatomy, physiology, electrocardiogram (ECG) and echocardiogram of the heart, all exclusively yielding positive results (Table 1).

Table 1. Summary of cardiovascular indices conforming to the golden proportions in the normal healthy state

- Ratio of vertical/transverse dimensions in LV (16)
- Ratios of LVEDd/LVESd and LVESd/(LVEDd–LVESd) (17)
- Length/width ratio in mitral anulus (16)
- Angle between outflow tract and continuity of inlet tract in RV (16)
- Angle between continuity of proximal ascending aorta and pulmonary trunk (16)
- Branching pattern and culprit lesion location along the course of coronary arteries (18, 19)
- Time-dependent maturation of diastolic functions during fetal life (20)
- Ratios of diastolic/systolic and R–R/diastolic time intervals (21)
- Night-time ratios of SBP/DBP and DBP/PP (22)

LV: left ventricle; LVESd and LVEDd: end-systolic and end-diastolic dimensions of the left ventricle, respectively; RV: right ventricle; R: R wave in electrocardiogram; DBP: lowest blood pressure in diastole; SBP: highest blood pressure in systole; PP: pulse pressure.

In accordance with this, Henein et al. Investigated the potential existence of Golden ratio and Golden angle in numerous cardiac and vascular parameters measured echocardiographically in their studies (16). The authors analyzed two diverse racioethnic groups (30 healthy Chinese subjects vs. 30 healthy Swedish vs.) with regard to their left ventricle (LV) transverse and vertical dimensions measured echocardiographically and made a comparison between them. As it was expected, both dimensions were smaller by 5 ± 0.5 mm and 8 ± 1 mm, respectively, in the Chinese group (16).

However, LV ratio (the ratio of the above dimensions) in both groups converged to a constant numerical value of 1.618, the Golden number implicated the greater importance of proportions than the absolute values of cardiac structures in achieving functional perfection of the heart (16). Beside this, the authors investigated whether LV ratio demonstrated a significant alteration in the setting of heart failure in correspondence to the severity of the disease. Patients with mild and with terminal-stage heart failure appeared to have LV ratio around 1.64 and 1.4 respectively. Probably due to extensive structural remodeling in the second group, this indicated

more significant impairment and resulted with more globoid LV pattern in these patients. Moreover, in patients with severe heart failure having the LV ratio significantly deviated from normal value (1.618), 3 years follow-up overall survival rates were only around 50%, suggesting the prognostic significance of Golden ratio in heart failure patients (16). Similarly, Yetkin E. et al. researched in a retrospectively designed study in a population of 1412 subjects with normal left ventricular ejection fraction (LVEF) values, whether left ventricular diameters represent the concept of golden ratio (17). Mean values of end-diastolic and end-systolic dimensions of the LV (LVEDd and LVESd, respectively) appeared to be 4.54 and 2.81 cm, respectively, through M-mode echocardiogram along with a mean value of LVEF measuring 68% (17). Further calculation and analysis of LVEDd/LVESd and LVESd/(LVEDd – LVESd) ratios showed mean numerical values of 1.614 ± 0.08 and 1.624 ± 0.21, respectively, both in perfect harmony with the Golden ratio (17). Mitral annulus is well known to build a fibrous skeleton having a resilient nature against tensile forces and provides a substantial quantity of sustenance for mitral valves and base of the heart. Nevertheless, maintenance of its normal dimensions and functions strongly depends on LV structure. Annular dimensions ratio in normal mitral valve was reportedly 1.6 and it is approximately close to Golden ratio (16). Recently, many studies regarding blood pressure and coronary arteries have confirmed that the human cardiovascular system serve as the reflection of divine aesthetics.

Conclusion

Number **ϕ** is not just an obscure term found in science of mathematics and physics. This Divine proportion reveals that our Universe was intelligently structured and at the same time it expresses a beauty of creation. It surrounds us in our everyday lives, even in our aesthetic criteria. The Golden ratio also appears in all modalities of nature and science, in human body in a variety of its functional and structural performances. However, there still exists a long way to go regarding diagnostic and prognostic implications, physiological and pathophysiological of Golden proportions and their wide use in clinical practice. The following researches still need to explore the potential relation between divine aesthetics and medicine along with the clear implications in clinical setting. Also, it is necessary to develop new methods in order to establish the divine reflection in our modifiable physiological functions expressed as measure of Golden proportion.

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MATEMATIČKA DETERMINISANOST U PRIRODI - ZLATNA PROPORCIJA

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Iako još uvek nije definisana deterministička jednačina života koju je predviđao Albert Anštajn, postoje jasno određeni tragovi i simboli matematičke pravilnosti koji se pojavljuju u prirodi, sa neverovatnom preciznošću. Jedan od takvih "Božanskih otisaka" u svetu koji nas okružuje je i Fibonačijev niz, koji možda predstavlja osnovu za otkrivanje neke univerzalne životne formule. Cilj rada bio je da objasni pojam Fibonačijevog niza i zlatnog preseka, kao i važnost otkrivanja postojećih savršenih matematičkih obrazaca u organskom i neorganskom svetu. Na taj način, moguće je jasnije i celovitije sagledati i razumeti fizičku, mentalnu i duhovnu prirodu čoveka.

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Ključne reči: *Fibonačijev niz, zlatni presek, anatomija, biologija, matematika*